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Towards a model of incoherent scatter signal spectra without averaging over sounding runs

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Abstract

This paper offers a model for incoherent scatter signal spectra without averaging the received signal over sounding runs (realizations). The model is based on the existent theory of radio waves single scattering from the medium dielectric permittivity irregularities, and on the existing kinetic theory of the plasma thermal irregularities. The proposed model is obtained for the case of monostatic sounding. The model shows that the main contribution to the received signal is made by ion-acoustic waves caused by certain spatial harmonics of the ions number phase density. The model explains the width and form of the signal spectrum by macroscopic characteristics of the medium, and its fine 'peaked' structure by characteristics of the ions number phase density. The notion of the weight volume is introduced to define the domain of wave vectors and velocities in the spatial spectrum of the ions number phase density which makes the main contribution to the formation of the scattered signal. This weight volume depends on the antenna pattern, the form of the sounding signal, and on the time window of spectral processing, as well as on ionospheric plasma macroscopic parameters: electron and ion temperatures, ion composition, and the drift velocity. Within the context of additional assumption about the ions number phase density, the proposed model was tested by the data from the Irkutsk incoherent scatter radar. The test showed a good fit of the model to experiment.

1 Introduction

One of the remote probing techniques for the ionosphere is the method of radio waves incoherent scatter. The method is based on the scattering of radio waves from ionospheric plasma dielectric permittivity irregularities [*Evans*, 1969]. Furthermore, two different experimental configurations are involved: monostatic (where the receive and transmit antennas are combined) and bistatic (where these antennas are spaced). In actual practice, it is customary to use the monostatic configuration. Ionospheric plasma parameters (ion composition, drift velocity, electron and ion temperatures, and electron density) in this case are determined from the scattered signal received after completion of the radiated pulse. The spectral power of the received signal, averaged over sounding runs ('realizations'), is related (assuming that such an averaging is equivalent to statistical averaging) to the mean spectral density of dielectric permittivity irregularities by the radar equation [*Tatarsky*, 1969]. The connection of the dielectric permittivity irregularities spectral density with mean statistical parameters of the medium is usually determined in terms of kinetic theory [*Clemow and Dougherty*,1969; *Sheffield*, 1975; *Kofman*,1997].

The location and size of the ionospheric region that makes a contribution to the scattered signal (sounding volume) is determined by the antenna beam shape, the sounding radio pulse, and by the time window of spectral processing [*Suni et al.*, 1989]. The shape of the sounding volume determines also the method's spectral resolution, the accuracy to which the mean spectral density of dielectric permittivity is determined (which, in turn, affects the determination accuracy of macroscopic ionospheric parameters: electron and ion temperatures, the drift velocity, and electron density). The number of realizations, over which the received signal spectral power is averaged, determines the method's time resolution, i.e. its ability to keep track (based on measurements) of fast changes of macroscopic parameters of ionospheric plasma.

Currently most incoherent scatter radars have accumulated extensive sets of the scattered signal individual realizations (private communications of P.Erickson (Millstone Hill), V.Lysenko (Kharkov IS radar), and G.Wannberg (EISCAT)). Therefore, attempts are made to analyze the realizations from different methods which differ from a standard averaging by their sounding runs. Basically, these methods imply looking for small scatterers making the main contribution to the scattered signal. This method is good for analyzing signals scattered from meteors and their traces [*Pellinen-Wannberg*, 1998]; however, it is insufficiently substantiated for describing the scattering in the ionosphere.

In the work there were used the experimental data obtained with Irkutsk Incoherent Scatter radar. The radar is located at $52^0 N, 104^0 E$, it has sounding frequency 152-160 MHz and peak power 3MW. High signal-to-noice ratio during the experiments under investigation (S/N ¿ 10) allows us to neglect the noice effects when analyzing the signal received.

The technique of the incoherent scatter signal processing in Irkutsk IS radar is the following. For each single realization of received signal we calculate spectrum in time window with width equal to the sounding signal duration and with delay corresponding to the radar range to the sounding volume investigated. The sounding signal we use in this experiment is a radiopulse with duration 800 mks. The repeating frequency approximately 25 Hz. Averaging over the 1000 realizations corresponds to 3% dispersium of the overaged spectrum relative to its mathematical expectation. The reason of using such a simple pulse is to investigate the fine structure of the single (unaveraged) spectrums in this simpliest case.

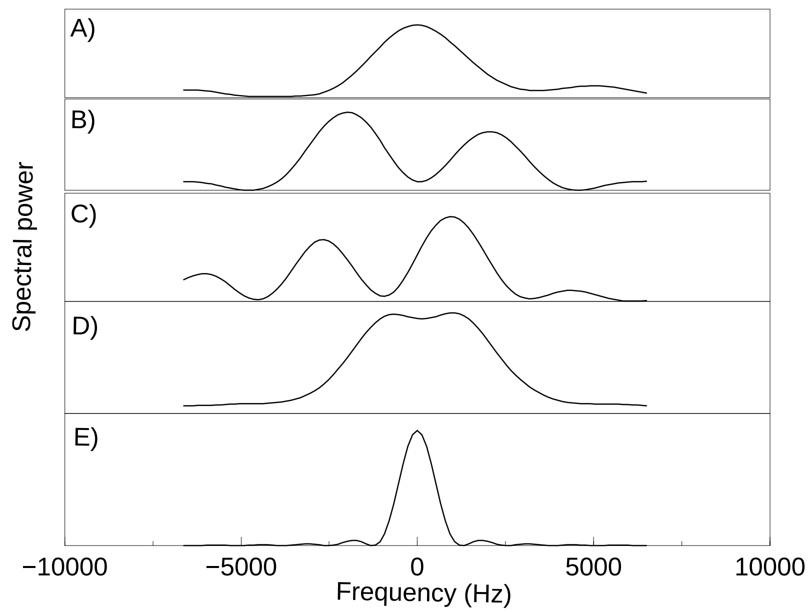

Figure 1: The spectral power of three different realizations (A-C), averaged over 1000 realizations spectral power of the scattered signal (D) and spectral power of the sounder signal envelope (E) as deduced using the data from the Irkutsk incoherent scatter radar.

Figure 1 exemplifies the mean spectral power of the scattered signal and its separate realizations, based on the data from the Irkutsk Incoherent Scatter radar. It is evident from the figure that the spectral power of the scattered signal in an individual realization (Figure 1(b-d)) differs drastically from that averaged over realizations (Figure 1(a)); therefore, existing model of the incoherently scattered signal, based on averaging over sounding runs, are inapplicable for its interpretation. For that reason, development of new models of the scattered signal for analyzing its separate realizations without averaging them is important from the theoretical and practical standpoint.

Sometimes it is useful to suppose that incoherent scattering signal is a random gaussian one [for example, *Farley*, 1969; *Zhou*, 1999]. But, it is well known that the signal received is a detirministic function of ionospheric dielectric permittivity ϵ and is fully determined in first approximation by the Born's formula (in one or another its form [*Ishimaru*, 1978; *Berngardt and Potekhin*, 2000]), this relation could be called as a radar equation for signals [*Berngardt and Potekhin*, 2000].

The dielectric permittivity irregularities also could be supposed as a random functions, but they are deterministic functional of some other functions (in case of uncollisional unmagnetized palsma with one ions type those functions are phase density of the ions and electrons as functions of velocity, location and time, ion composition and temperatures of the ions and electrons, this functional dependence is determined by the Landau's solution [*Landau*, 1946]).

If one could determine all these unknown functions, the received signal shape in single realization will be fully determined, and could be analyzed without using any statistical methods. Such an approach, for example, is used in radioacoustical technique of the athmosphere sounding when the delectric permittivity irregularities (by which the radisignal is scattered) are generated by the acoustical wave [*Kalistratova and Kon*, 1985].

The statistical proporties of the single realizations are showed at Figure 2. From this figure it becomes clear that the unaveraged spectrum has the fine structure - it consists from a number of peaks with approximately 1.5KHz width (and this width very slightly depends on frequency), which could be characterized by the peak amplitude(amplitude at the maximum of the peak) and peak appearence (number of realizations in which there is a peak maximum at given frequency) at the given frequency, and those properties distributions are not gaussian ones but have double peaked structure and located in the same band with incoherent scattering average spectral power.

This fact allows as to suppose that not only average spectral power of the received signal depends on ionosperical parameters, but the fine structure of non-averaged spectra too.

At first, it is neccessary to understand quilitatively, what information one could obtain from one realization of the IS signal. It is well known, that after any statistical processing of a function a part of the information is loosed irreversibly (for example. when one calucaltes the first n statistical moments, all the rest moments, starting with n+1 are still unknown). That is why, if the statistical characteristics of the realizations (mean spectral power or correlation function) are depend on the ion and electron temperatures and the ion composition then single realization must depend on all those parameters and on some new 'additional' parameters. It is clear that to determine temperatures and ion composition from averaged signal parameters is much easier than from single realization (because the second one includes additional parameters), and we can use the ones obtained from mean spectral power, with necessary spatial and spectral resolution, using different techniques, for example alternating codes [*Lehtinen*, 1986]. But the new 'additional' parameters can be determined from single realizations only.

The aim of this paper is to find out the functional dependence of single realization spectrum on all the parameters, including well known (temperatures and ion composition) and new ones, which could describe the single realizations spectrum properties. For this propose we will use for analysis only signals with high signal to noise ratio (more than 10), because in this case the noice effects could be neglected and the received signal could be supposed as only IS signal without presence any noice.

2 Initial expressions

To analyze the individual realizations of the scattered signal, it is necessary to have a convenient expression relating the spectrum of the scattered signal to the space-time spectrum of dielectric permittivity irregularities without averaging over realizations. Such an expression for a monostatic experimental configuration was obtained and analyzed in [*Berngardt and Potekhin*, 2000]. It holds true in the far zone of the receive-transmit antenna and, within constant factors (unimportant for a subsequent discussion), is

$$u(\omega)=\int H(\omega-\nu,k-2k_0-\frac{\nu}{2c})\frac{g(-\widehat{\mathrm{k}})}{k}\widetilde{\epsilon}(\nu,\overrightarrow{k})d\nu d\overrightarrow{k}. \tag{1}$$

Here $\widetilde{\epsilon}(\nu,\overrightarrow{k})$ - is the space-time spectrum of dielectric permittivity irregularities; $H(\omega,k)\approx\int H(t,r)e^{-i(\omega t+kr)}drdt=\int o(t)a(t-2r/c)e^{-i(\omega t+kr)}dtdr/r$ - is the narrow-band weight function; $a(t),o(t)$ - are, respectively, the sounder signal envelope and the time window of spectral processing; $g(\widehat{r})$ - is the antenna factor which is the product of the antenna patterns by reception and transmission; $\widehat{r}=\overrightarrow{r}/r$ - is a unit vector in a given direction; k_0 - the wave number of the sounding wave; c -is the velocity of light.

Suppose that sounding signal and receiving window of spectral processing are located in time near the moments $t=T_1-T_0$ and $t=T_1$ respectively and theirs carriers do not intersects (this is the one of the radar equation (1) obtaining conditions [*Berngardt and Potekhin*, 2000]). In this case the carrier of the weight function $H(t,r)$ is located near the $t=T_1; r=T_0c/2$. By going in equation (1) to the spectrums calcualated relative to the weight volume center (to remove the oscillated multipliers under integral), we obtain (neglecting to the unessentional multiplier):

$$\begin{array}{rl} u(\omega)= & \int H_1(\omega-\nu,k-2k_0-\frac{\nu}{2c})\frac{g(-\widehat{\mathrm{k}})}{k} \\ & \widetilde{\epsilon}(\nu,\overrightarrow{k};T_1-T_0/2,-\widehat{k}T_0c/2)d\nu d\overrightarrow{k} \end{array}. \tag{2}$$

where $H_1(\omega,k)\approx H(\omega,k)e^{ikT_0c/2}$ - low oscillating part of the $H(\omega,k)$, corresponding to its calculation relative to the center of the weight volume $H(t,r)$; and $\widetilde{\epsilon}(\nu,\overrightarrow{k};T,\overrightarrow{R})=\widetilde{\epsilon}(\nu,\overrightarrow{k})e^{i(\nu T-\overrightarrow{k}\overrightarrow{R})}$ - is a time-spatial spectrum of dielectric permittivity irregularities calculated relative to the point $t=T$, $\overrightarrow{r}=\overrightarrow{R}$.

In accordance with [*Sheffield*,1975; *Clemmow and Dougherty*,1969; and *Akhieszer et al.*,1974], assume that the spectrum of small-scale dielectric permittivity irregularities is determined by the Landau solution [*Landau*,1949]. Then the low-frequency (ion-acoustic) part of the irregularities spectrum in a statistically homogeneous, unmagnetized, collisionless ionospheric plasma with one sort of ions is determined by plasma macroscopic parameters (electron and ion temperatures, ion composition, and drift velocity), and by unknown conditions in the moment T related to which this spectrum is calculated - the ions number phase density in a six-dimensional phase space of velocities and positions of particles. It is known that the dielectric permittivity irregularities spectrum at large wave numbers of the sounding wave $k_0 > \omega_N/c$ (where ω_N is plasma frequency) , is proportional to the electron density irregularities spectrum [*Landau and Lifshitz*,1982, par.78]:

$$\widetilde{\epsilon}(\omega, \overrightarrow{k}; T) = -\frac{4\pi q_e^2}{k_0^2 m_e c^2} n_{e1}(\omega, \overrightarrow{k}; T),$$

which is given by the expression (for example, [*Sheffield*, 1975, sect.6]):

$$n_{e1}(\omega, \overrightarrow{k}; T) = \frac{G_e(\omega, \overrightarrow{k})}{\epsilon_{||}(\omega, \overrightarrow{k})} \int \frac{exp(i\overrightarrow{k}\overrightarrow{r}) f_{i1}(\overrightarrow{r}, \overrightarrow{v}; T)}{\omega - \overrightarrow{k}\overrightarrow{v} - i\gamma} d\overrightarrow{r} d\overrightarrow{v}, \qquad (3)$$

where $\epsilon_{||}(\omega, \overrightarrow{k}) = \left(1 + G_e(\omega, \overrightarrow{k}) + G_i(\omega, \overrightarrow{k})\right)$ - is longitudinal dielectric permittivity; wave number k should be small enought to wave length be smaller than Debye length (Solpiter approximation). Most part of IS radars have the sounding frequencies(50-1000 MHz) within these limitations.

$$G_{e,i}(\omega, \overrightarrow{k}) = \frac{4\pi |q_{e,i} q_e| n_{e,i0}}{m_{e,i} k^2} \int\limits_{-\infty}^{+\infty} \frac{\overrightarrow{k} \frac{\partial f_{0e,i}}{\partial \overrightarrow{v}}}{\omega - \overrightarrow{k}\overrightarrow{v} - i\gamma} d\overrightarrow{v}; \qquad (4)$$

$f_{e,i0}(\overrightarrow{v})$, $n_{e,i0}$- are equilibrium distribution functions of the electrons and ions velocity and their densities; $m_{e,i}, q_{e,i}$- are the mass and charges of electrons and ions, respectively;

$$f_{i1}(\overrightarrow{r}, \overrightarrow{v}; T) = \sum_{j=1}^{N} \delta(\overrightarrow{r} - \overrightarrow{r_j}(T)) \delta(\overrightarrow{v} - \overrightarrow{v_j}(T)) - f_{i0}(\overrightarrow{v}) \qquad (5)$$

- the ions number phase density in a six-dimensional phase space of velocities and positions of particles (the ions number phase density, INPD, at $t = T$), with the summation made over all ions. Generally equilibrium distribution functions $f_{e,i0}(\overrightarrow{v})$ are taken to be Maxwellian, with the temperatures T_e and T_i for electrons and ions, respectively, and in the absence of a drift they are

$$f_{e,i0} = exp(-(v/V_{T_{e,i}})^2)/(\pi V_{T_{e,i}}^2)^{3/2},$$

where $v_{T_{e,i}} = (2kT_{e,i}/m_{e,i})^{1/2}$ stands for the thermal velocities of electrons and ions, respectively. Then the functions $G_{e,i}(\omega, \overrightarrow{k})$ have the well-known analytical expression, for example [*Sheffield*,1975]:

$$G_{e,i}(\omega, \overrightarrow{k}) = \left(\frac{1}{k\lambda_D}\right)^2 \frac{q_{e,i}T_e}{q_e T_{e,i}} \left(Rw(x_{e,i}) - iIw(x_{e,i})\right) \qquad (6)$$

where

$$x_{e,i} = \omega/(kv_{T_{e,i}});$$

$$Rw(x) = 1 - 2xe^{-x^2} \int_0^x e^{p^2} dp$$

$$Iw(x) = \pi^{1/2} xe^{-x^2}$$

The physical meaning of the expression (3) is as follows: the position and velocity of each ion at the moment T are determined by the INPD $f_{i1}(\overrightarrow{r}, \overrightarrow{v}; T)$, and the dielectric permittivity irregularities $\widetilde{\epsilon}(\omega, \overrightarrow{k}; T)$ are determined by ion-acoustic oscillations of plasma under the action of such initial conditions.

3 Traditional processing of the incoherent scatter signal, and characteristics of its separate realizations

Traditionally, the incoherent scattered signal is processed in the following way. A set of the scattered signal spectra (1) is used to obtain its spectral power averaged over realizations. By assuming that an averaging over the realizations is equivalent to a statistical averaging, and also by assuming a Maxwellian distribution of the INPD $f_{i1}(\overrightarrow{r}, \overrightarrow{v}; T)$, one can obtain the following expression for the mean spectral power of the scattered signal [*Suni et al.*,1989]:

$$< |u(\omega)|^2 > \approx \frac{const}{R^2} \frac{2\pi q_i}{kq_e} \int F(\omega - \nu) \left| \frac{G_e(\nu, 2k_0)}{\epsilon_{||}(\nu, 2k_0)} \right|^2 f_{i0}\left(\frac{\nu}{2k_0}\right) d\nu \qquad (7)$$

where $F(\omega)$ is the 'smearing' function determined by the spectrum of the sounder signal and the spectral processing time window; and $<>$ is averaging over realizations.

The frequency dependence of the scattered signal mean spectral power (7) under usual ionospheric conditions has a typical 'two-hump' form (Figure 1(a)) [*Evans*,1969]. From the scattered signal mean spectral power (7) it is possible to determine the electron T_e and ion T_i temperatures and the drift velocity $\overrightarrow{v}_0$ involved in a familiar way in the functions $\epsilon_{||}(\omega, 2k_0)$, $G_e(\omega, 2k_0)$ and $f_{i0}(v)$ [*Sheffield*,1975].

In single realizations, however, the scattered signal spectral power differs essentially from the mean spectral power. Figure 1 presents the scattered signalspectral power in three consecutive realizations (Figure 1(A-C)), the spectral power averaged over 1000 realizations (Figure 1(D)), and the spectral power of the sounder signal envelope (Figure 1(E)). From Figure 1 it is evident that the non averaged spectral power of the incoherent scatter signal (Figure 1(A-C)) has a typical 'peaked' form, the width of peaks is larger than that of the sounder

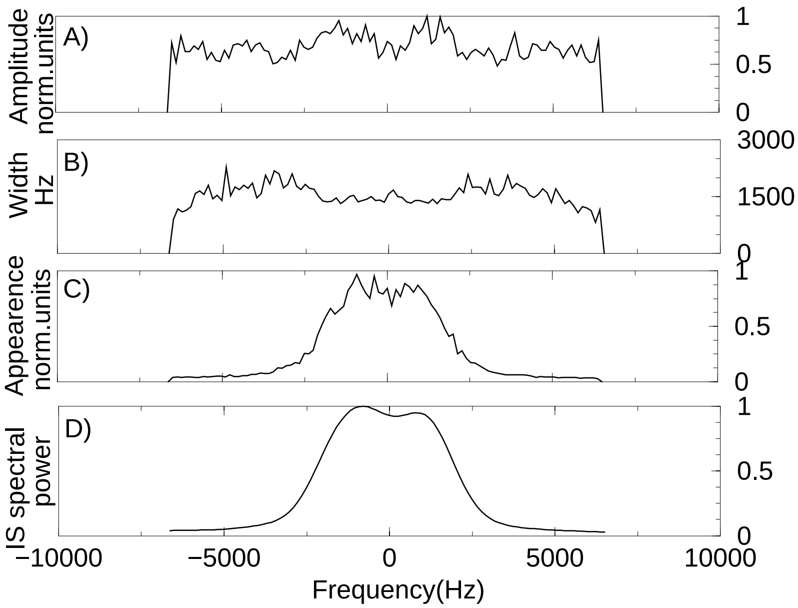

Figure 2: Statistical proporties of single spectrum realizations (averaged over 1000 realizations) as functions of frequency. Mean peak amplitude (A), mean peak width(B) and mean appearence of the peaks (C) for different frequencies of received signal. For comparison there is showed the IS spectral power for this data (D).

signal spectrum, and the peaks themselves are concentrated in the band of the mean signal spectral power. In the case of an averaging over realizations, such a peaked structure transforms to a typical smooth two-hump structure (Figure 1(D)).

4 Model of single realizations of incoherent scatter signals

4.1 Structure of one realization of the incoherent scatter signal spectrum

To obtain a model of the incoherent scatter signal we substitute the Landau's expression for dielectric permittivity irregularities (3) into the expression for the scattered signal spectrum (2). Using in (3) the spatial spectrum $\widetilde{f_{i1}}(\overrightarrow{k},\overrightarrow{v};T)$ of the ions number phase density $f_{i1}(\overrightarrow{r},\overrightarrow{v};T)$, and upon interchanging the order of integration, we obtain the one-realization model for the scattered signal spectrum:

$$u(\omega) = \int K(\omega,\overrightarrow{k},v_{||})F_{i1}(\overrightarrow{k},v_{||};T_1 - T_0/2)d\overrightarrow{k}dv_{||}, \tag{8}$$

$$K(\omega,\overrightarrow{k},v_{||}) = \frac{g(-\widehat{k})}{k^3}\int \xi(\nu,\overrightarrow{k})\frac{H(\omega-\nu,k-2k_0-\nu/c)}{(\nu-kv_{||}-i\gamma)}d\nu. \tag{9}$$

Here

$$F_{i1}(\overrightarrow{k},v_{||};T) = \int \widetilde{f}_{i1}(\overrightarrow{k},\overrightarrow{v};T)\delta(\overrightarrow{k}\overrightarrow{v}-kv_{||})d\overrightarrow{v} \tag{10}$$

is unknown function we want to determine from experiment and has a form similar to the Radon transform of the function $\widetilde{f}_{i1}(\overrightarrow{k},\overrightarrow{v};T)$. A kinetic function (showed at Figure 3)

$$\xi(\nu,\overrightarrow{k}) = G_e(\nu,\overrightarrow{k})/\epsilon_{||}(\nu,\overrightarrow{k}) \tag{11}$$

is determined by macroscopic parameters of ionospheric plasma $T_{e,i}$ and $\overrightarrow{v_0}$; these parameters can be determined, for example, from measurements of the mean spectral power of the received signal (7).

Thus the kernel $K(\omega,\overrightarrow{k},v_{||})$ is completely determined by the sounder signal, the receiving window, and by macroscopic characteristics of ionospheric plasma. The expression (8) clearly shows the meaning of the kernel $K(\omega,\overrightarrow{k},v_{||})$: it determines the selective properties of the model, i.e. the possibilities of determining the unknown function $F_{i1}(\overrightarrow{k},v_{||};T)$ from the measured $u(\omega)$. Hence it can be termed the weight volume in the space $(\overrightarrow{k},v_{||})$, or ambiguity function. Since the function $H_1(\omega,k)$ is a narrow-band one, with its carrier concentrated near zero, the function of indefiniteness $K(\omega,k,v_{||})$ has also a limited carrier in k near

$k = 2k_0$. The possibilities of determining the unknown function $F_{i1}(\overrightarrow{k}, v_{||}; T)$ dependence on the wave vector directions $\widehat{k}$ are determined by the product of the kinetic function $\xi(\omega, \overrightarrow{k})$ and the antenna beam $g(-\widehat{k})$.

According to the resulting model (8), the form of scattered signal single spectrum is determined both by a determinate component (the weight volume $K(\omega, k, v_{||})$), and by a random (i.e. dependent on time by the unknown way) component. A random component is the function $F_{i1}(\overrightarrow{k}, v_{||}; T)$ determined by the spartial harmonics packet of the INPD $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ with wave numbers k, concentrated near $2k_0$ and calculated relative to the moment $t = T$. The moment T is determined by the moments of the sounding signal transmitting and spectral processing receiving window location, and corresponds to the middle moment between them $T = T_1 - T_0/2$. The weight volume $K(\omega, k, v_{||})$ determines the parameters of this wave packet, the region of wave vectors and velocities in $F_{i1}(\overrightarrow{k}, v_{||}; T)$ which make the main contribution to the scattered signal at a particular frequency ω.

4.2 Qualitative properties of the weight volume

In this experiment the spectra sounder signal $a(\omega)$ and the receiving window $o(\omega)$ are selected such as they are sufficiently narrow-band ones (1KHz), in comparison with the functions $\epsilon_{||}(\omega, 2k_0), G_e(\omega, 2k_0)$ and $f_{i0}(v)$, in order to improve the accuracy of their determination from experimental results. Therefore, the weight function $H_1(\omega, k) \sim o(\omega - ck/2)a(ck/2)$ can also be considered narrow-band from both arguments as compared with the kinetic function $\xi(\nu, \overrightarrow{k})$ from corresponding arguments. In this case we can approximate the weight volume $K(\omega, \overrightarrow{k}, v_{||})$ as:

$$K(\omega, \overrightarrow{k}, v_{||}) \approx \frac{\xi(\omega, 2k_0\widehat{k})g(-\widehat{k})}{k^3} \int \frac{H_1(\omega - \nu, k - 2k_0 - \nu/c)}{(\nu - kv_{||} - i\gamma)} d\nu. \qquad (12)$$

The function $H_1(\omega, k)$ is concentrated near $\omega = 0, k = 0$ [*Berngardt and Potekhin*,2000]. The width of this function from arguments (ω, k) is $\Delta\omega = (\Delta\omega_a + \Delta\omega_o)$, $\Delta k = (\Delta\omega_a + \Delta\omega_o)/c$, where $\Delta\omega_a, \Delta\omega_o$, is the width of bands of sounder signal spectra and of the spectral processing window, respectively. In this experiment on ionospheric sounding by the incoherent scatter method, they have the order $\Delta\omega_{a,o} \sim 10^4 sec^{-1}$. The function $\xi(\nu, \overrightarrow{k})$ at a fixed $k = 2k_0 = 6.28m^{-1}$ for a typical ionospheric plasma with O^+ ions and $T_e = T_i = 1500K$ is presented in Figure 3. The figure shows that the function $\xi(\nu, \overrightarrow{k})$ does varies smoothly on the characteristic size of the weight function $H_1(\omega, k)$ carrier in ω which is in this case has the order of $\Delta\Omega \sim 10^4 sec^{-1}$ (corresponds to a sounding by the impulse radio signal of a duration of 1 millisecond).

Assuming that the envelope of the sounder pulse $a(t)$ and the receiving window $o(t)$ have an identical Gaussian-like spectrum: $a(\omega) = o(\omega) = exp(-(\omega/\Delta\omega)^2)$, we obtain the function $H_1(\omega, k)$ of the form

$$H_1(\omega, k) = Be^{-((\omega - kc/2)/\Delta\omega)^2} e^{-(kc/(2\Delta\omega))^2}. \qquad (13)$$

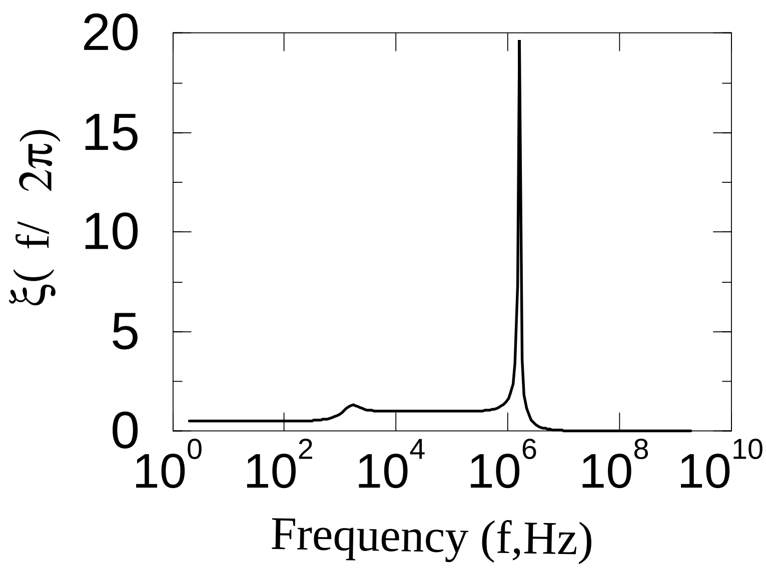

Figure 3: Kinetic function $\xi(\nu, \vec{k})$ as a function of ν at a fixed $k = 2k_0 = 6.28 m^{-1}$ for a typical ionospheric plasma with O^+ ions and $T_e = T_i = 1500K$.

Upon substituting (13) into (12) and rather unwieldy calculations, similar to [*Landau*, 1946], we obtain the following expression for the scattered signal spectrum (8):

$$u(\omega) = \int K(\omega, \overrightarrow{k}, v_{||}) \widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T_1 - T_0/2) dv_{||} d\overrightarrow{k}, \tag{14}$$

where

$$K(\omega, \overrightarrow{k}, v_{||}) = V_1(\omega - kv_{||}) V_2(\omega - (k - 2k_0)c) V_3(\omega, \widehat{k}) \tag{15}$$

$$\begin{aligned} V_1(\omega) &= i\pi e^{-\Phi^2(\omega)} - Rw(\Phi(\omega)); \\ V_2(\omega) &= e^{-\Phi^2(\omega)}; \\ V_3(\omega, \widehat{k}) &= \tfrac{\xi(\omega, 2k_0\widehat{k}) g(-\widehat{k})}{8k_0^3} \end{aligned} \tag{16}$$

$$\Phi(\omega) = \frac{\omega}{\sqrt{2}\Delta\omega} \tag{17}$$

The selective properties of $K(\omega, \overrightarrow{k}, v_{||})$ in the longitudinal component of the velocity $v_{||}$ are determined by the first cofactor V_1 in (15). A maximum $K(\omega, \overrightarrow{k}, v_{||})$ in $v_{||}$ at a fixed ω is determined by the condition $V_1(\omega - kv_{||,0}) = max$ which, as a consequence of the properties of the exponential and the Rw functions, corresponds to the frequency Doppler shift condition in the case of the scattering from a single particle:

$$v_{||,0} = \frac{\omega}{k}. \tag{18}$$

The width $\Delta v_{||}$ of a maximum $K(\omega, \overrightarrow{k}, v_{||})$ in $v_{||}$ (that determines the region of velocities making the main contribution to the scattered signal at fixed ω and $\overrightarrow{k}$) can be estimated from the condition $\Phi(\omega - kv_{||,0} \pm \Delta v_{||}) = 1$ to be

$$\Delta v_{||} = \sqrt{2}\Delta\omega/k. \tag{19}$$

The selective properties of $K(\omega, \overrightarrow{k}, v_{||})$ in wave numbers k are determined by the second cofactor V_2 in (15). A maximum $K(\omega, \overrightarrow{k}, v_{||})$ in k, at a fixed ω, is determined by the condition $V_2(\omega - (k - 2k_0)c) = max$ which corresponds to the condition (analogical to the Volf-Bragg condition for scattering from nonstationary spatial harmonic):

$$k = 2k_0 + \frac{\omega}{c}. \tag{20}$$

The width Δk of a maximum $K(\omega, \overrightarrow{k}, v_{||})$ in k (that determines the region of wave numbers making the main contribution to the scattered signal at a fixed ω) can be estimated from the condition $\Phi(\omega - (k - 2k_0)c \pm \Delta kc) = 1$ to be

$$\Delta k = \sqrt{2}\Delta\omega/c. \tag{21}$$

The function V_3 determines the selective properties of $K(\omega, \overrightarrow{k}, v_{||})$ in the direction of the wave vectors $\widehat{k}$ and the maximum possible width of the received signal spectrum determined by the kinetic function $\xi(\omega, 2k_0\widehat{k})$ and antenna factor $g(-\widehat{k})$.

5 Discussion

From (14) it follows that the fine structure of the scattered signal spectrum $u(\omega)$ is determined only by the properties of the function $\widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T)$ (10) and can be related to the INPD $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ only within the framework of additional assumptions about the structure of $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$. Formally, to determine the function $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ require measuring the scattered signal for different wave numbers k_0 of the sounder signal simultaneously.

To carry out a qualitative comparison with experimental spectra of incoherently-scattered signals we use the spectral processing window that repeats the sounder signal shape. Their spectra are approximated by Gaussian-like spectra with a width equal to their actual spectrum width. According to (14), the spectrum of the received signal is defined by the unknown function $\widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T)$(10) (convoluted with a kernel $K(\omega, \overrightarrow{k}, v_{||})$). For comparison with experimental data, we give following simple model of the function $\widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T)$.

Simple model Assume that $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ involves only one peak in the range of longitudinal wave vectors and velocities of our interest:

$$\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T) = \delta(\overrightarrow{k} - \overrightarrow{k_1})\delta(\overrightarrow{v} - \overrightarrow{v_1}).$$

$$\widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T) = \delta(\overrightarrow{k} - \overrightarrow{k_1})\delta(\overrightarrow{k}\overrightarrow{v_1} - kv_{||}).$$

This model corresponds to the fact that the medium involves an isolated spatial harmonic $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ at the wave vector $\overrightarrow{k_1}$ and with longitudinal velocity v_1, the amplitude of which is significantly larger than the amplitudes of spatial harmonics close to it. The spectrum of the received signal will then involve also only one peak:

$$u(\omega) \sim V_1(\omega - k_1 v_1) V_2(\omega - (k_1 - 2k_0)c) V_3(\omega, \widehat{k_1}), \tag{22}$$

and the form and width of this peak will be defined by the product $V_1(\omega - k_1 v_1)V_2(\omega - (k_1 - 2k_0)c)$. From the position of the peak in the spectrum $u(\omega)$, one can determine the unknown wave number k_1 of the spatial harmonic $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$ that make the main contribution to the scattered signal (20) and the longitudinal velocity v_1 (18) corresponding to the moment T .

This model gives the signal spectrum with only one peak. According to the model, the observed presence of several peaks in the real spectrum corresponds to existance not only one peculiarity described by the model but a numer of them.

Let us show that after averaging this model gives as the well known expression for the IS average spectral power. According traditional approaches lets suppose that $< f_{i1}(\vec{k}, \vec{v}, T) >= f_{i0}(\vec{v})(when|k| > 0)$. Summarizing over the different realizations gives the produces the function:

$$
\begin{aligned}
< |u(\omega)|^2 >\sim\ & \int \left|V_3(\omega,\widehat{k})\right|^2 d\widehat{k} \sum_{j=1}^{N} f_{i0}(v_{||,j}) \\
& \left|V_1(\omega - 2k_0 v_{||,j})\right|^2 \int |V_2(\omega-(k-2k_0)c)|^2\, dk \\
& = const \left|\int V_3(\omega,\widehat{k}) d\widehat{k}\right|^2 \int f_{i0}(v)\, |V_1(\omega-2k_0 v)|^2\, dv \\
& \sim \left|\frac{G_e(\omega,2k_0)g(-\widehat{k_0})}{8k_0^3\epsilon_{||}(\omega,2k_0)}\right|^2 \int f_{i0}(v)\, |V_1(\omega-2k_0 v)|^2\, dv
\end{aligned}
\tag{23}
$$

From (23) becomes clean, that the average received signal spectral power $< |u(\omega)|^2 >$ is determined as a product of the kinetic function $|\xi(\omega, 2k_0\widehat{k})|^2$ and maxwellian ions distribution $f_{i0}(v)$ convolved with a narrow-band function $|V_1(\omega - 2k_0 v)|^2$ (which is defined by the sounding signal and spectral processing receiving window spectra). Qualitatively this solution (23) is close to the average spectral power obtainded by the tradditional way (7).

A numerical simulation have been made for the model containing a number of peaks:

$$
\widetilde{F}_{i1}(\overrightarrow{k}, v_{||}; T) = \delta(sin(kA + \phi(T)) * sin(B(v_{||}) + \phi(T)) - 1), \tag{24}
$$

where A - some constant; $B(v) \sim \int_0^v exp(-x^2/v_{Ti}^2)dx$; $\phi(T)$ - a function with uniform distribution of values (we have used random one, having fixed value for fixed T). This expression for F_{i1} has a normal distribution over $v_{||}$, uniform distribution over k, and statistical independence of the values for different T (or when delay between them exceeds the interval in which $\phi(T)$ changes slowly). Spectra obtained by substituting this simple model (24) into obtained equation (8), gives us the following single spectrums and their statistical properties, are shown at Figure 4. As one can see, single realizations of the spectral power(E-G) have the same structure, as an experimental one (Figure 1), the close peak width (B) (Figure 2,B), and same relation between mean peak appearence (C) and average spectral power (D) (Figure 2,C,D). This allows us to suppose that the model of the scattered signal single spectrum (8) could be used to describe signal properties, and simplified model (24) could quilitatively describe the ion number phase density behavior.

6 Conclusion

In this paper we have suggested an interpretation of separate realizations of incoherently scattered signal spectra. It is based on the radar equation [*Berngardt and Potekhin*,2000] and kinetic theory of ion-acoustic oscillations of a statistically homogeneous unmagnetized, collisionless ionospheric plasma with one sort of ions [*Landau*,1946].

In accordance with the proposed model (8), the main contribution to the scattering is made by plasma waves caused by spatial harmonics of ions number phase density $\widetilde{f}_{i1}(\overrightarrow{k}, \overrightarrow{v}; T)$, with wave numbers on the order of the double wave number of the sounder signal $k \approx 2k_0$, for $T = T_1 - T_0/2$.

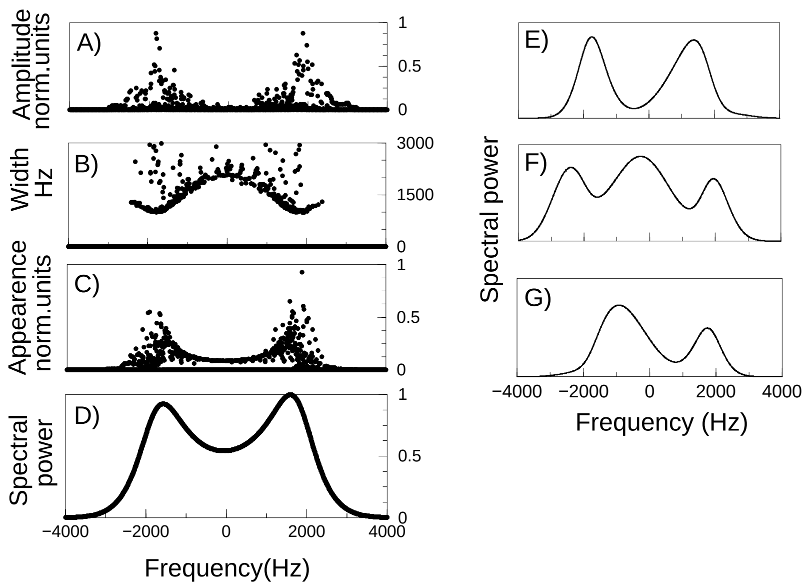

Figure 4: Numerical simulation results for $T_e = T_i = 1500K$ using equations (8), (15) and simple model (24). Mean peak amplitude (A),mean peak width(B), mean peak appearence(C), average spectral power (D) and three spectral realizations (E-G).

It has been shown that the form of the received signal spectrum $u(\omega)$ is related to the INPD $\widetilde{f}_{i1}(\overrightarrow{k},\overrightarrow{v};T)$. At each frequency ω the value of $u(\omega)$ is determined by Radon's-like integral $\widetilde{F}_{i1}(\overrightarrow{k},v_{||};T)$ on $\widetilde{f}_{i1}(\overrightarrow{k},\overrightarrow{v};T)$ between the limits of the velocity components across the wave vector $\overrightarrow{k}$ (10). The region of wave vectors and longitudinal velocities making contribution to the received signal $\overrightarrow{k},v_{||}$ is determined by the weight volume $K(\omega,\overrightarrow{k},v_{||})$ (9).

So, by changing the transmitting pulse start moment and the moment of its receiving one could measure the value of $\widetilde{F}_{i1}(\overrightarrow{k},v_{||};T)$, as function of the time T. This allows the INPD $\widetilde{f}_{i1}(\overrightarrow{k},\overrightarrow{v};T)$ diagnostics for different moments including delays smaller than theirs lifetime and without statistical averaging of receiving signal. Actually, in the case of irregularities lifetime much longer than sounding pulses repeating interval $T_{i+1}-T_{i}$., one could measure the $\widetilde{F}_{i1}(\overrightarrow{k},v_{||};T_i-T_0/2)$ behavior as function of the time $t=T_i-T_0/2$.

Based on the proposed model in (8) and a Gaussian approximation of the spectra of the sounder signal envelope and the receiving window, a qualitative comparison of the model with experimental data from the Irkutsk incoherent scatter radar was carried out. The comparison showed a quilitative agreement for simplified model (24), based on additional assumptions about the properties of the function $\widetilde{F}_{i1}(\overrightarrow{k},v_{||};T)$.

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